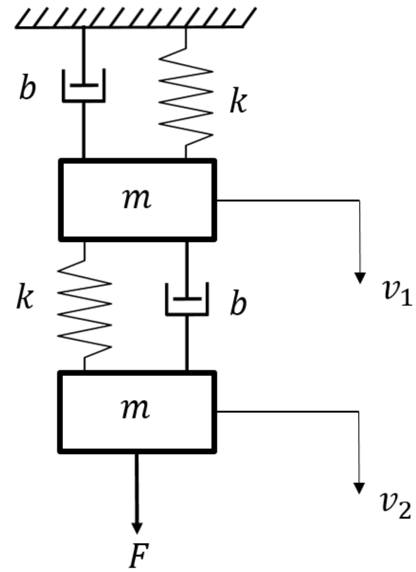
Homework 4

Question 1

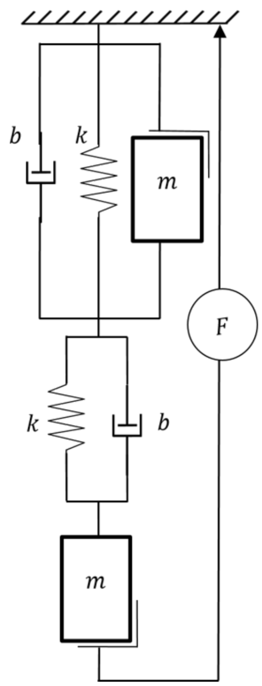
For the system shown below,

a) Draw the **Mechanical Circuit**
b) Draw the **Impedance Circuit**
c) Determine the **Mobility Equation**

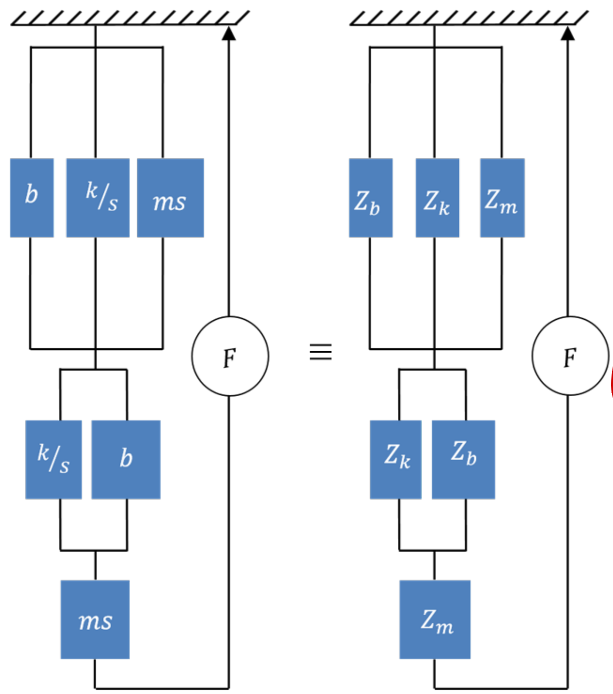
Note: the impedance equation <u>must be simplified</u>



Solution



Mechanical Circuit



Impedance Circuit

$$M_{eq} = \frac{1}{Z_{eq}} = \frac{s}{ms^2 + bs+k} + \frac{s}{bs+k} + \frac{1}{ms}$$

$$\rightarrow \quad M_{eq} = \frac{(bs+k_2)(ms) + (ms^2 + bs+k)(ms) + (ms^2 + bs+k)(bs+k)}{(ms^2 + bs+k)(bs+k)(ms)}$$

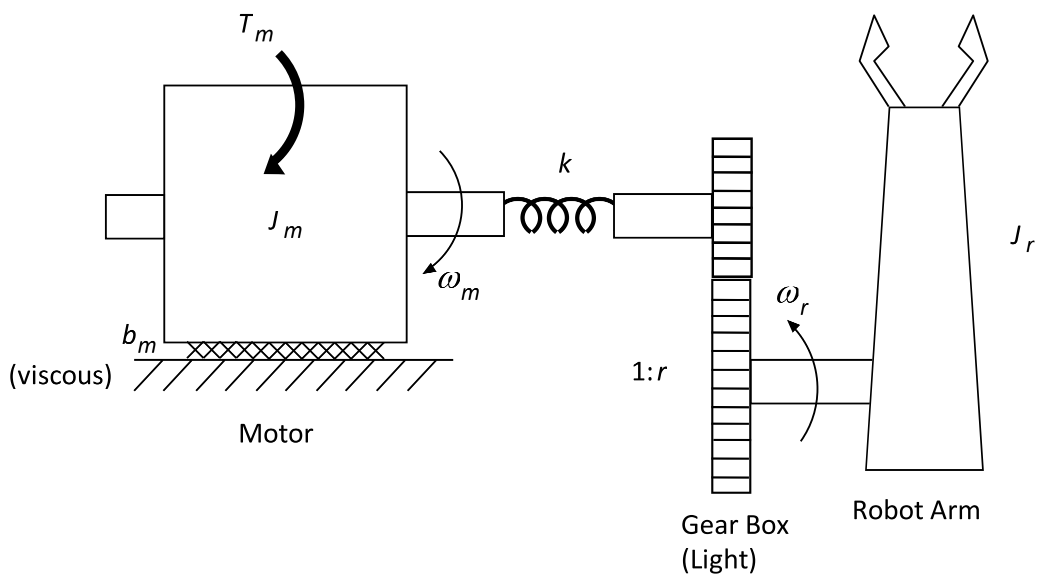
$$\rightarrow \quad M_{eq} = \frac{m(m+b)s^3 + (mk + 2mb + b^2)s^2 + 2k(m+b)s + k^2}{(m^2 b)s^4 + m(mk + b^2)s^3 + 2mbks^2 + (mk^2)s}$$

Question 2

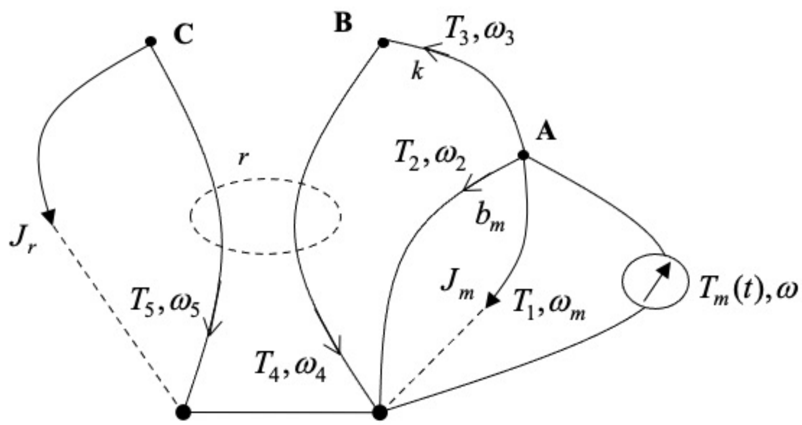
A model for a single joint of a robotic manipulator is shown in Figure below. The usual notation is used. The gear inertia is neglected and the gear reduction ratio is taken as $1{:}r$ (*Note*: $r < 1$).

a) Draw a linear graph for the model, assuming that no external (load) torque is present at the robot arm.

b) Using the linear graph derive a state model for this system. The input is the motor magnetic torque T_m and the output is the angular speed ω_r of the robot arm. What is the order of the system?



(a) The linear graph is shown below.



(b) This is a 3rd order model. The 3 energy-storage elements are independent, even though a gear box is present.

Constitutive equations:

$$\left.\begin{aligned} J_m \frac{d\omega_m}{dt} &= T_1 \\ \frac{dT_3}{dt} &= k\omega_3 \\ J_r \frac{d\omega_r}{dt} &= T_6 \end{aligned}\right\} \text{State Space Shell}$$

$$T_2 = b_m \omega_2$$

$$\omega_5 = r\omega_4$$

$$T_5 = -\frac{1}{r} T_4$$

Continuity equations (node equations):

$$\boldsymbol{A}: \; -T_m + T_1 + T_2 + T_3 = 0$$

$$\boldsymbol{B}: \; -T_3 + T_4 = 0$$

$$\boldsymbol{C}: \; T_5 + T_6 = 0$$

Compatibility equations (loop equations):

$$-\omega + \omega_m = 0$$

$$-\omega_m + \omega_2 = 0$$

$$-\omega_m + \omega_3 + \omega_4 = 0$$

$$-\omega_5 + \omega_r = 0$$

Eliminate unwanted variables:

$$T_1 = T_m - T_2 - T_3 = T_m - b_m \omega_2 - T_3 = T_m - b_m \omega_m - T_3$$

$$\omega_3 = \omega_m - \omega_4 = \omega_m - \frac{\omega_5}{r} = \omega_m - \frac{\omega_r}{r}$$

$$T_6 = -T_5 = \frac{T_4}{r} = \frac{T_3}{r}$$

By substitution into the shell, we get the following state equations:

$$J_m \frac{d\omega_m}{dt} = T_m - b_m\omega_m - T_3$$

$$\frac{dT_3}{dt} = k(\omega_m - \frac{\omega_r}{r})$$

$$J_r \frac{d\omega_r}{dt} = \frac{T_3}{r}$$

Now with: State $\boldsymbol{x} = \begin{bmatrix} \omega_m & T_3 & \omega_r \end{bmatrix}^T$; Input $\boldsymbol{u} = \begin{bmatrix} T_m \end{bmatrix}$; Output $\boldsymbol{y} = \begin{bmatrix} \omega_r \end{bmatrix}$

we have:

$$\boldsymbol{A} = \begin{bmatrix} -b_m/J_m & -1/J_m & 0 \\ k & 0 & -k/r \\ 0 & 1/(rJ_r) & 0 \end{bmatrix}; \qquad \boldsymbol{B} = \begin{bmatrix} 1/J_m \\ 0 \\ 0 \end{bmatrix}$$

$$\boldsymbol{C} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}; \boldsymbol{D} = 0$$